12.26. Model: Model the planet and satellites as spherical masses.

Visualize: Please refer to Figure Ex12.26.

Solve: (a) The period of a satellite in a circular orbit is $T = [(4\pi^2/GM)r^3]^{1/2}$. This is independent of the satellite's mass, so we can find the ratio of the periods of two satellites a and b:

$$\frac{T_{\rm a}}{T_{\rm b}} = \sqrt{\left(\frac{r_{\rm a}}{r_{\rm b}}\right)^3}$$

Satellite 2 has $r_2 = r_1$, so $T_2 = T_1 = 250$ min. Satellite 3 has $r_3 = (3/2)r_1$, so $T_3 = (3/2)^{3/2}T_1 = 459$ min. (b) The force on a satellite is $F = GMm/r^2$. Thus the ratio of the forces on two satellites a and b is

$$\frac{F_{\rm a}}{F_{\rm b}} = \left(\frac{r_{\rm b}}{r_{\rm a}}\right)^2 \left(\frac{m_{\rm a}}{m_{\rm b}}\right)^2$$

Satellite 2 has $r_2 = r_1$ and $m_2 = 2m_1$, so $F_2 = (1)^2 (2)F_1 = 20,000$ N. Similarly, satellite 3 has $r_3 = (3/2)r_1$ and $m_3 = m_1$, so $F_3 = (2/3)^2 (1)F_1 = 4440$ N.

(c) The speed of a satellite in a circular orbit is $v = (GM/r)^2$, so its kinetic energy is $K = \frac{1}{2}mv^2 = GMm/2r$. Thus the ratio of the kinetic energy of two satellites a and b is

$$\frac{K_{\rm a}}{K_{\rm b}} = \left(\frac{r_{\rm b}}{r_{\rm a}}\right) \left(\frac{m_{\rm a}}{m_{\rm b}}\right)$$

Satellite 3 has $r_3 = (3/2)r_1$ and $m_3 = m_1$, so $K_1/K_3 = (3/2)(1/1) = 3/2 = 1.50$.